# Letters

## Comments on "Calculation of Cutoff Wavenumbers for TE and TM Modes in Tubular Lines with Offset Center Conductor"

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In the paper[1] by Vishen *et al.* [1], the authors refer to two papers of ours, numbered [8] and [10] in their list of references, and apparently without any elaboration make certain unwarranted comments on the method and results of [8], namely: "Some of the tabulated parameters of [8] actually yield nonphysical results; also, symmetric and antisymmetric modes appear to be degenerate, which contradicts experimental observations."

Before refuting these remarks, let us point out certain errors and misstatements of the article. In [1, eq. (1)], the authors define $k$ as the wavenumber of the medium. It is actually $k_c = (\omega^2\mu\epsilon - \beta^2)^{1/2}$. Otherwise, above cutoff the transverse $(r, \theta$ or $x, y)$ dependence of the field of the mode becomes a function of frequency, whereas it is solely, through $k_c$, a function of the cross-sectional geometry of the guide. This is not a simple typographical error, because the authors explicitly define $k$ as $\omega\sqrt{\mu\epsilon}$, the wavenumber.

A misstatement also appears in the last paragraph of their introduction. They claim a rigorous mathematical derivation starting from the Helmholtz equation (with $k$ instead of $k_c$!). Their method, however, in no way differs from the approach followed by many authors: separation of variables and use of the well-known addition theorems for Bessel functions to satisfy the boundary condition at the second circular boundary. The infinite set of linear equations obtained in this way is the same in all papers, including ours; no one can claim originality at this point.

They proceed by solving the set numerically, a procedure requiring repetition each time the eccentricity $d$ changes. In [8] we chose to solve it analytically for *small* eccentricities $k_c d$, preserving a generality—inherent in all analytical methods—that dispenses with the need to solve numerically a large set of equations each time $d$ changes. How restricted our results are because of this limitation is discussed in detail in [8] and is shown to provide very good results for appreciable $k_c d$. This is further corroborated by the results of Vishen *et al.*, which are in very good agreement with our results from [8] as long as $k_c d$ is not too large. Table I here compares results from Table I (TE modes) in Vishen *et al.*, Kuttler's lower and upper bounds, and our own from [8]. The agreement for $k_c d < 1$ (sometimes even for $k_c d > 1$) is remarkable. In certain cases, our results are within Kuttler's bounds, whereas those of Vishen *et al.* (marked by an asterisk) fall outside. Also, missing roots in Vishen *et al.* (marked by a bar) are supplied by our method; so, in the third case ($n = 0.5$, $d = 0.2$) our result is $k_{c41} = 5.1059$ and falls very near Kuttler's bounds. Vishen *et al.* miss this zero. In general, they fail to provide any kind of explanation for these two exceptional cases (asterisks and bars.) It is also obvious that our results start to deviate only when $k_c d$ exceeds 1. This is, also, the prevailing case for the values of Table II (TM modes) in [8] and no meaningful comparison was possible. Even there, the tendency to obtain similar results was obvious as $k_c d$ became smaller.

TABLE I
CUTOFF WAVENUMBERS FOR TE MODES

| | | Symmetric Modes | | | | | Antisymmetric Modes | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Kcmn | Vishen et al | Kuttler's bounds Lower | Upper | Ours from [8] Kcmn | Kcd | Vishen et al | Kuttler's bounds Lower | Upper | Ours from [8] Kcmn | Kcd |
| $n = \frac{2}{3}$, d=0.2 | 11 | 1.2522* | 1 32027 | 1.32221 | 1.30227 | 0 242 | 1 1917 | 1 19001 | 1 19176 | 1.19524 | 0 242 |
| | 21 | 2 4365 | 2 4408 | 2 4446 | 2 43136 | 0 482 | 2 4307* | 2 4267 | 2 4305 | 2 43136 | 0 482 |
| | 31 | 3 6209 | 3 6157 | 3 6218 | 3 6180 | 0 722 | 3.6203 | 3 6142 | 3 6203 | 3 6180 | 0 722 |
| | 41 | 4 7897 | 4.7804 | 4 7901 | 4 78866 | 0 960 | 4.7896 | 4.7804 | 4 7899 | 4.78866 | 0 960 |
| | 51 | 5 9379 | 5 9218 | 5 9385 | 5 9370 | 1 195 | 5 9379 | 5 9231 | 5.9385- | 5 9370 | 1 195 |
| $n = \frac{1}{3}$, $d = \frac{2}{9}$ | 11 | 1 5619* | 1 5766 | 1 5807 | 1 57903 | 0 342 | 1 5435 | 1 5393 | 1 5436 | 1 54373 | 0 342 |
| | 21 | 2 9064 | 2 8968 | 2 9067 | 2 90555 | 0 651 | 2 9058 | 2.8966 | 2 9067 | 2 90555 | 0 651 |
| | 31 | 4 1152 | 4 0944 | 4 1161 | 4 10792 | 0 925 | 4 1152 | 4 0955 | 4 1161 | 4 10792 | 0 925 |
| | 41 | 4 4220* | 4 2146 | 4 2356 | 5 26892 | 1 179 | 5.1606 | 5 131 | 5 167 | 5 26892 | 1 179 |
| | 51 | 5.2669 | 5.219 | 5.270 | 6 3957 | 1 425 | 5 2758 | 5.237 | 5 279 | 6 3957 | 1.425 |
| n=0.5, d=0 2 | 11 | 1.3793* | 1.40694 | 1 40793 | 1.40408 | 0 271 | 1.3522 | 1.35114 | 1.35219 | 1 35278 | 0 271 |
| | 21 | 2 6849 | 2 6837 | 2 6862 | 2.6844 | 0 536 | 2 6838 | 2 6815 | 2 6840 | 2 6844 | 0 536 |
| | 31 | 3 9295 | 3 9247 | 3 9298 | 3.9288 | 0 792 | 3.9296 | 3 9247 | 3.9298 | 3 9288 | 0.792 |
| | 41 | - | 4 9937 | 5 0192 | 5 1059 | 1 035 | 5 1131 | 5.1036 | 5 1138 | 5 1059 | 1 035 |
| | 51 | 5 1131 | 5 1031 | 5.1139 | 6.24515 | 1 268 | 5.8106 | 5 793 | 5 834 | 6 24515 | 1 268 |

In view of all these data, it is evident how unwarranted are their comments on the nonphysical nature of our results and the (contrary to observation!) "degeneracy" in symmetric and antisymmetric modes. They, also, obtain the same results. Compare, for instance, their values $k_{cn1}$ $(n = 3,4,5)$ for symmetric and antisymmetric TE modes. They are almost identical, exhibiting the same "degeneracy" as ours. In our case, degeneracy, if it exists, is due to the limitation $k_c d \ll 1$ and is restricted to order $(k_c d)^2$. It is obvious from our theory that even and odd $k_c$ would differ if terms of order $(k_c d)^4$ could be maintained. The results of Vishen *et al.*, being so close to ours, actually verify this conclusion.

Finally, in Table II (TM modes) in Vishen *et al.*, the results for the antisymmetric modes should be moved down one line to restore the correct correspondence with the results of the symmetric modes on the left.

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[1]A. Vishen, G. P. Srivastava, G. S. Singh, and F. Gardiol, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 292–294, Feb. 1986.

## Additional Comments[2]

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Although Vishen *et al.* refer to the early work of Veselov and Semenov [6], no comparison is made between their formulation and the one found in [6]. I fail to recognize any essential differences between the two approaches. Furthermore, the authors state that the technique "based on Graf's addition theorem for Bessel functions" was developed by Singh and Kohtari [2], whereas in fact it appears that the same technique was employed in [6] considerably earlier.

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